Logicality and the Picture Theory of Language

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Grammar and logic

- Grammar distinguishes between sentences and non-sentences

(1) a. John smokes → sentence
b. smokes smokes → non-sentence

- Logic distinguishes between valid and non-valid arguments

(2) a. $\dfrac{\text{John smokes but Mary doesn't}}{\text{John smokes}}$ → valid argument
b. $\dfrac{\text{Mary doesn't smoke}}{\text{John smokes but Mary doesn't}}$ → invalid argument

- Wittgenstein's 1921 claim: (1) and (2) are one and the same phenomenon

PTL

- Sentencehood and validity are to be accounted for by one unified theory which
  - tells us what a sentence is
  - tells us, for any sentence ϕ, which other sentences are true if ϕ is true
  - dissolves all philosophical problem
- This is the "picture theory of language" (PTL)

4.01	Der Satz ist ein Bild der Wirklichkeit [...]	A proposition is a picture of reality [...]
3.21	Der Konfiguration der einfachen Zeichen im Satzzeichen entspricht die Konfiguration der Gegenstände in der Sachlage.	The configuration of simple signs in a propositional sign corresponds to the configuration of objects in a state of affairs.

→ English translation by Michael Beaney.

An illustration

	Sentences of L_F	Sentences of L_W	States of affairs described
1	$S(j)$		John smokes
2	$\neg S(m)$		Mary doesn't smoke
3	$S(j) \wedge \neg S(m)$		John smokes but Mary doesn't

- Syntactic rules
  - needed by L_F to exclude gibberish such as $S(S)$, $j(m)$, $j(S)$, etc.
  - not needed by L_W: gibberish are ineffable.
- Logical rules
  - needed by L_F to guarantee e.g. $\phi \wedge \psi \Rightarrow \phi$
  - not needed by L_W: entailment is shown

Stenius (1960), Hope (1965)

5.13	Dass die Wahrheit eines Satzes aus der Wahrheit anderer Sätze folgt, ersehen wir aus der Struktur der Sätze.	That the truth of one proposition follows from the truth of other propositions can be seen from the structure of the propositions.
5.132	"Schlussgesetze", welche – wie bei Frege und Russell – die Schlüsse rechtfertigen sollen [...] wären überflüssig.	"Laws of inference", which are supposed – e.g. by Frege and Russell – to justify inferences [...] would be superfluous.
5.473	Die Logik muss für sich selber sorgen [...]	Logic must take care of itself [...]

A prediction of PTL

- There is no way to translate $S(j) \wedge \neg S(j)$ into $\mathrm{L_W}$: it is not possible to place the pebble inside and outside of the jar
- Conjecture: if there is no translation of ϕ, there is no translation of $\neg\phi$

(3) Prediction of PTL
Trivialities are non-sentences

- At several places Wittgenstein seems to be aware of this result

4.462	Tautologie und Kontradiktion sind nicht Bilder der Wirklichkeit [...]	Tautology and contradiction are not pictures of reality [...]
4.466	[...] Sätze, die für jede Sachlage wahr sind, können überhaupt keine Zeichenverbindungen sein [...]	[...] propositions that are true for every state of affairs cannot be combinations of signs at all [...]

A distinction without a difference?

- However, Wittgenstein stops short of considering trivialities non-sentences, introducing instead an obscure distinction
  - Non-sentences are non-sensical (unsinnig)
  - Trivialities are senseless (sinnlos)

4.4611	Tautologie und Kontradiktion sind aber nicht unsinnig; sie gehören zum Symbolismus [...]	Tautology and contradiction [...] are not nonsensical; they are part of the symbolism [...]
4.461	[...] Tautologie und Kontradiktion sind sinnlos [...]	[...] Tautology and contradiction are senseless [...]

An inconsistency

- Wittgenstein seems to make the following three claims

  (A) Sentences are pictures
  (B) Trivialities are not pictures
  (C) Trivialities are sentences

- Why commit to (C)?
- I conjecture that the reason has a phenomenological and a logical component

Intuition and analysis

- Wittgenstein perceives (4a) and (4b) as well-formed sentences

(4) a. It's raining and not raining
b. It's raining or not raining

- He analyzes them as (5a) and (5b)

(5) a. $p \wedge \neg p$
b. $p \vee \neg p$

PTL: Recap

- Wittgenstein considers trivialities to be well-formed, even though his theory (PTL) predicts them to be ill-formed
- I will now present a perspective under which
  - Trivialities are ill-formed
  - Sentences which Wittgenstein analyzes as trivial are not really trivial

$\rightarrow$ The perspective in question goes by the name of "Logicality"

The hypothesis

Universal Grammar interfaces with a natural deductive system and filters out sentences expressing trivialities

cf. Chierchia (2006), Del Pinal (2019, 2022)

Some background

- I assume a standard semantics for quantifiers

(6) a. $\text{every}(P)(Q) = 1$ iff $P \subseteq Q$
b. $\text{a}(P)(Q) = 1$ iff $P \cap Q \neq \emptyset$

(7) a. $\text{every}(\text{student})(\text{smokes}) = 1$ iff $\text{student} \subseteq \text{smokes}$
b. $\text{a}(\text{student})(\text{smokes}) = 1$ iff $\text{student} \cap \text{smokes} \neq \emptyset$

Cf. Barwise and Cooper (1981), Heim and Kratzer (1998)

Example 1: exceptives

(8) a. every student but John came
b. #a student but John came

- Analysis: the exception must be minimal (von Fintel 1993)

(9) D(P but E)(Q) = 1 if and only if
a. D(P−E)(Q) = 1
b. If D(P−C)(Q) = 1 then E ⊆ C, for any C

- Explanation: (8b) is ill-formed because it is contradictory

(10) a. (8a) ⇒ every student who is not John came and it is not the case that every student came
b. (8b) ⇒ a student who is not John came and it is not the case that a student came

Example 2: existentials

(11) a. there is a fly in my soup
b. #there is every fly in my soup

- Analysis: *there is* relates its associate with U by way of D

(12) there is(D(P)) = 1 iff D(P)(U) = 1

- Explanation: (11b) is ill-formed because it is tautological

(13) a. (11a) $\Leftrightarrow \{x \mid x \text{ is a fly in my soup}\} \cap U \neq \emptyset$
b. (11b) $\Leftrightarrow \{x \mid x \text{ is a fly in my soup}\} \subseteq U$

cf. Milsark (1977), Barwise and Cooper (1981)

Other phenomena

- The body of works carried out under the perspective of Logicality is large and growing.
  - mass vs. count nouns (Chierchia 1998, 2010)
  - individual-level vs. stage-level predicates (Magri 2009)
  - the distribution of polarity items (Krifka 1995, Chierchia 2013, Crnič 2019)
  - free choice (Menéndez-Benito 2005, Crnič and Haida 2020)
  - numerals (Bylinina and Nouwen 2018, Haida and Trinh 2020, 2021)
  - comparatives (Gajewski 2008)
  - island effects (Fox and Hackl 2006, Abrusán 2007)
  - question embedding (Uegaki and Sudo 2017)
  - ...

A Tractarian problem for Logicality

- We say that (14a) is ill-formed because it has the inconsistent entailment in (14b). But why is (13b) well-formed?

(14) a. #a student but John came
b. a student who is not John came and it is not the case that a student came

- More dramatically: why is (15) well-formed?

(15) It's raining and not raining

Solution: contextualism

- Natural language grammar contains a covert, context-sensitive "rescaling" operator, R_c, which attaches to non-logical expressions and modulates their meaning

(16) It's raining and not raining

a. raining $\wedge$ $\neg$raining $\rightarrow$ trivial

b. R_c(raining) $\wedge$ $\neg R_{c'}$(raining) $\rightarrow$ contingent

(17) #A student but John came

a. a(student but John)(came) $\rightarrow$ trivial

b. a(R_c(student) except $R_{c'}$(John))($R_{c''}$(came)) $\rightarrow$ trivial

Cf. Del Pinal (2019), Pistoia-Reda and Sauerland (2021), Del Pinal (2022)

An open issue

- What is a logical constant?
  - Wittgenstein (1921): logical constants do not represent
  - Gajewski (2003): logical constants are permutation-invariant
- there are unsolved problems...

Cf. MacFarlane (2017), Abrusán (2019), Del Pinal (2022)

Wittgenstein and homophony

- Wittgenstein did not know about Logicality, hence never considered the possibility that he was confused by homophony in cases such as (18)

  (18) it's R_c(raining) and not $R_{c'}$(raining)

- Interestingly, he did discuss homophony, albeit in light of a more transparent example

3.323 In der Umgangssprache kommt es ungemein häufig vor, dass dasselbe Wort auf verschiedene Art und Weise bezeichnet – also verschiedene Symbolen angehört [...] Im Satze "Grün ist grün" – wo das erste Wort ein Personenname, das letzte ein Eigenschaftswort ist – haben diese Worte nicht einfach verschiedene Bedeutung, sondern es sind verschiedene Symbole [...]

In everyday language it occurs extremely often that the same word signifies in different ways – that is, belongs to different symbols [...] In the proposition 'Green is green' – where the first word is a person's name, the last an adjective – these words do not simply have different meaning but involve different symbols [...]

Logicality: Recap

- Trivialities are ill-formed
- Seemingly well-formed and trivial sentences can be analyzed as contingent by way of rescaling

A parallel

- Linguists in 2023:
  - Every well-formed sentence is
    - generated by syntactic rules
    - contingent
- Wittgenstein in 1921:
  - Every well-formed sentence is
    - a truth-function of elementary propositions
    - pictorial

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