

Logicality and the abstraction approach to natural language

Tue Trinh

Leibniz-Zentrum Allgemeine Sprachwissenschaft

Umass Amherst
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Abstraction Thesis

[T]he notion 'grammatical' cannot be identified with 'meaningful' or 'significant' in any semantic sense [...] (1) and (2) are equally nonsensical, but any speaker of English will recognize that only the former is grammatical.

- (1) Colorless green ideas sleep furiously.
- (2) Furiously sleep ideas green colorless.

Chomsky (1957: 15)

→ 'abstraction' borrowed from Pistoia-Reda (2024)

Round square

Man darf [...] das Sinnlose (das Unsinnige) nicht zusammenwerfen mit dem Absurden (dem Widersinnigen), welches [...] ein Teilgebiet des Sinnvollen ausmacht. Die Verknüpfung 'ein rundes Viereck' liefert wahrhaft eine einheitliche Bedeutung [...]. Sagen wir hingegen 'ein rundes oder; 'ein Mensch und ist'; u. dgl., so existieren gar keine Bedeutungen [...].

[...] [O]ne must not confound the senseless (or nonsensical) with the absurd (or counter-sensical) [...] [which] is rather a sub-species of the significant. The combination 'a round square' really yields a unified meaning [...] But if we say 'a round or', 'a man and is' etc., there exist no meanings [...].

Husserl (1901: 312)

Analytic philosophy

Analytic philosophy presupposes Abstraction, as it claims that language can deceive us into thinking that a well-formed expression is meaningful while in fact it is meaningless.

Bewitchment

[...] Die Philosophie ist ein Kampf gegen die Verhexung unsres Verstandes durch die Mittel unserer Sprache.

[...] Philosophy is a struggle against the bewitchment of our understanding by the resources of our language.

Wittgenstein (1953)

It is plain that the grammatical structure of our everyday language can justly be charged with being misleading [...]. [I]t [is] possible within natural language to form meaningless but grammatically correct sentences [...].

Dummett (1973)

Scheinsätze

Im strengen Sinn sinnlos ist [...] eine Wortreihe, die innerhalb einer bestimmten, vorgegebenen Sprache gar keinen Satz bildet. Es kommt vor, daß eine solche Wortreihe auf den ersten Blick so aussieht, als sei sie ein Satz; in diesem Falle nennen wir sie einen Scheinsatz. Unsere These behauptet nun, daß die angeblichen Sätze der Metaphysik sich durch logische Analyse als Scheinsätze enthüllen.

Carnap (1931)

In the strict sense [...] a sequence of words is meaningless if it does not, within a specified language, constitute a statement. It may happen that such a sequence of words looks like a statement at first glance; in that case we call it a pseudo-statement. Our thesis, now, is that logical analysis reveals the alleged statements of metaphysics to be pseudo-statements.

Questions

- (A) How can language 'deceive' us with well-formed expressions if there is no presupposition that what is well-formed is also meaningful?
- (B) What does it mean to say that this presupposition is erroneous?

Frequency

Suppose we say we think a well-formed expression is meaningful because most well-formed expressions are meaningful.

- (A) How do we know this? Through logical analysis?
- (B) There would be infinitely many well-formed but meaningless expressions.
- (C) Does frequency matter at all for judgement of grammaticality or meaningfulness?

Chomsky (1957)

Privilege of syntax

Why should we assume that (A) makes sense but (B) does not?

- (A) We know this expression is well-formed, but we need logical analysis to know whether it's meaningful.
- (B) We know this expression is meaningful, but we need syntactic analysis to know whether it's well-formed.

Apodictic evidence

Is A more similar to B or to C?

- (A) I don't know if the sentence I'm looking at is really meaningful.
- (B) I don't know if the house I'm seeing is really red.
- (C) I don't know if the color I'm seeing is really red.

Husserl (1901, 1907)

Identity Thesis

Well-formedness is meaningfulness.

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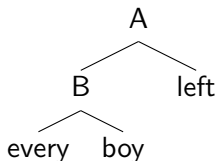
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Truth condition

The meaning of every expression is identified as the truth condition of a sentence in which it occurs.

(1)



a. $\llbracket \text{every} \rrbracket = \lambda P. \lambda Q. \forall x. P(x) \rightarrow Q(x)$

b. $\llbracket \text{boy} \rrbracket = \lambda x. x \text{ is a boy}$

c. $\llbracket \text{left} \rrbracket = \lambda x. x \text{ left}$

d. $\llbracket B \rrbracket = \llbracket \text{every} \rrbracket (\llbracket \text{boy} \rrbracket) = \lambda Q. x \text{ is a boy} \rightarrow Q(x)$

e. $\llbracket A \rrbracket = \llbracket B \rrbracket (\llbracket \text{left} \rrbracket) = 1 \text{ iff } \forall x. x \text{ is a boy} \rightarrow x \text{ left}$

Heim and Kratzer (1998)

Frege

Nach der Bedeutung der Wörter muss im Satzzusammenhange, nicht in ihrer Vereinzelung gefragt werden.

Frege (1884: x)

Nur im Zusammenhange eines Satzes bedeuten die Wörter etwas.

Frege (1884: §62)

Never to ask for the meaning of a word in isolation, but only in the context of a proposition.

[I]t is only in the context of a proposition that words have any meaning [...].

Wittgenstein

[...] Es ist unmöglich, daß Worte in zwei verschiedenen Weisen auftreten, allein und im Satz.

[...] It is impossible for words to occur in two different ways, alone and in the proposition.

Wittgenstein (1921: 2.0122)

Nur der Satz hat Sinn; nur im Zusammenhang des Satzes hat ein Name Bedeutung.

Only the proposition has sense; only in the context of a proposition has a name meaning.

Wittgenstein (1921: 3.3)

Carnap

Im strengen Sinn sinnlos ist [...] eine Wortreihe, die innerhalb einer bestimmten, vorgegebenen Sprache gar keinen Satz bildet.

In the strict sense [...] a sequence of words is meaningless if it does not, within a specified language, constitute a statement.

Carnap (1931)

Non-assertive speech acts

It has been argued that speech acts are represented in the grammar. To the extent that the argument is convincing, every sentence is a statement.

- (2) A: Is it raining? [A ASK [whether it is raining]]
B: It is. [B ASSERT [it is raining]]

Stenius (1967), Ross (1970), Lakoff (1970), Gazdar (1979), Chomsky (1981, 1986, 2001), Krifka (2001), Miyagawa (2012), Krifka (2015), Sauerland and Yatsushiro (2017), Trinh and Truckenbrodt (2018), Krifka (2019, forthcoming), Trinh (2022), Wiltschko (2021), Miyagawa (2022), Trinh and Bassi (2023), Trinh (2024a,b), Fox (2024), Fox et al. (2024)

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Grammar

The fundamental aim in the linguistic analysis of a language L is to separate the grammatical sequences which are sentences of L from the ungrammatical sequences which are not sentences of L and to study the structure of the grammatical sequences. The grammar of L will thus be a device that generates all of the grammatical sequences of L and none of the ungrammatical ones. We thus face a familiar task of explication of some intuitive concept – in this case, the concept ‘grammatical in English’, and more generally, the concept ‘grammatical’.

Chomsky (1957: 13)

Derivation

We define a 'derivation' as a finite sequence of strings, beginning with [...] Σ , and with each string in the sequence being derived from the preceding string by application of one of the instruction formulas of F .

Chomsky (1957: 29).

- (3) Σ : S
F: (i) $S \rightarrow NP + VP$
(ii) $NP \rightarrow D + N$, John, Mary
(iii) $VP \rightarrow V + NP$
(iv) $D \rightarrow$ the, every, some
(v) $N \rightarrow$ boy, girl
(vi) $V \rightarrow$ saw, helped

Grammaticality

A rarely noted fact: Chomsky's (1957) grammar generates only sentences.

- (4) S
- | | |
|--------------------------|-------|
| NP + VP | (i) |
| D + N + VP | (ii) |
| D + N + V + NP | (iii) |
| every + N + V + NP | (iv) |
| every + boy + V + NP | (v) |
| every + boy + saw + NP | (vi) |
| every + boy + saw + Mary | (ii) |
- (5) #NP
- | | |
|-------------|------|
| D + N | (ii) |
| every + N | (iv) |
| every + boy | (v) |

A puzzle about phrases

But there does seem to be well-formedness contrast between phrases.

- (6) a. Intuitively well-formed:
every boy, saw Mary, a round square
- b. Intuitively ill-formed:
every Mary, boy saw, a round or

Part of a sentence

If well-formedness were simply understood as being part of a sentence, then 'a round or' would be well-formed, since 'this is a round or elliptical table' is certainly a sentence.

Bar-Hillel (1957: 366)

Resolution: ellipsis

- (A) Only sentences are grammatical
 - (B) Sentences can be pronounced in full or in part
 - (C) A sentence *S* can be pronounced in part if the silent portion of *S* is the background of an (implicit or explicit) question which *S* answers
-
- (7)
 - a. every boy ~~saw Mary~~ ← who **saw Mary**?
 - b. ~~John~~ saw Mary ← what did **John** do?
 - c. every boy ~~saw Mary~~ ← #?
 - d. a round square ~~doesn't exist~~ ← what doesn't exist?
 - e. ~~this is a round or elliptical table~~ ← #?

Minimalism

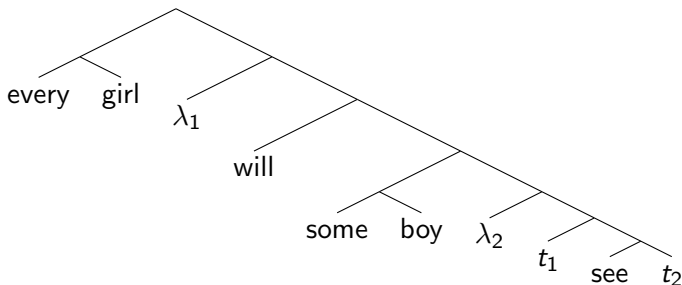
In the more recent ‘minimalist’ variant of generative syntax, an expression is well-formed if its derivation ‘converges’, i.e. if the output of that derivation is a (sentential) structure which contains nothing but semantically relevant information.

Chomsky (1995, 2000), Radford (2004), Carnie (2006), i.a.

Minimalist derivation

Numeration: some_{F₁}, every_{F₂}, boy_{F₃}, girl_{F₄}, see_{F₅}, will_{F₆}

Logical Form:



where each F_n is a set of ‘formal’ – i.e. ‘uninterpretable’ – features such as category, subcategory, type, case, agreement, EPP, etc.

Communication

Users of language(s) can be charitable: they can guess what the speaker wants to communicate from a faulty expression that is strictly speaking uninterpretable by the rules of the system.

- (8) a. I will talk to her.
b. #Me will talk to she.

- (9) a. $(p \rightarrow (q \rightarrow p))$
b. $\#(p \rightarrow (q \rightarrow p))$

Linguistic theory is consistent with Identity: every well-formed expression is a meaningful expression and vice versa.

Can we go home?

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Meaningful but ungrammatical

There are intuitively ill-formed expressions which are derivable in the best theory of syntax and interpretable in the best theory of semantics.

- (10) a. Every boy but John left
 $\approx$ every boy who is not John left $\wedge \neg$ every boy left
- b. #Some boy but John left
 $\approx$ some boy who is not John left $\wedge \neg$ some boy left $= \perp$
- (11) a. At least two boys left
 $=$ two boys left $\vee$ more than two boys left
- b. #At least zero boys left
 $=$ zero boys left $\vee$ more than zero boys left $= \top$

von Fintel (1993), Haida and Trinh (2020)

Logicity Thesis

Trivialities are excluded by grammar.

Gajewski (2002), Abrusán (2007), Chierchia (2013), Abrusán (2019),
Pistoia-Reda and San Mauro (2021), Pistoia-Reda (2024), i.a.

Tractarian problem

The explanation of the ungrammaticality of these sentences should itself be ungrammatical.

- (12) a. #some boy but John left
 b. some boy who is not John left and it's not the case that some boy left
- (13) a. #at least zero boys left
 b. zero boys left or more than zero boys left

And what about sentences such as those in (14)?

- (14) a. it's raining and not raining
 b. if it's raining, it's raining

The superengineer

Problem: contexts are infinite, phonemes are finite. Solution: infinitely many silent indices.

- (15)
- a. she_c is $tall_c$
 - b. every $student_{c'}$ is $tall_{c'}$
 - c. not every $students_{c''}$ is $tall_{c''}$

Once the superengineer has provided the machinery, nothing prevents speakers from making 'creative' use of it.

- (16)
- a. $John_c$ is both a $student_{c'}$ and not a $student_{c''}$
 - b. if it's raining $_c$, it's raining $_{c'}$

Dissolving the Tractarian problem

When indices do not help, ungrammaticality ensues.

- (17) a. #some boy_c but John_{c'} left_{c''}
 b. some boy_c who is not John_{c'} left_{c''} and it's not the case that
 some boy_{c'''} left_{c''''}

Syntax–phonology mapping

In der Umgangssprache kommt es ungemein häufig vor, dass dasselbe Wort auf verschiedene Art und Weise bezeichnet – also verschiedene Symbolen angehört [...] Im Satze “Grün ist grün” – wo das erste Wort ein Personennamen, das letzte ein Eigenschaftswort ist – haben diese Worte nicht einfach verschiedene Bedeutung, sondern es sind verschiedene Symbole [...]

In everyday language it occurs extremely often that the same word signifies in different ways – that is, belongs to different symbols [...] In the proposition ‘Green is green’ – where the first word is a person’s name, the last an adjective – these words do not simply have different meaning but involve different symbols [...]

Wittgenstein (1921: 3.323)

- Q: Can we maintain Identity in light of Logicity?
- A: No! Because some meaningful expressions are non-sentences.
- Q: Is there a way to think about grammar, i.e. about what a sentence is, such that trivialities cannot be sentences?
- A: Maybe.

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The Tractatus

The Tractatus is an attempt to go beyond Frege's logicist program, i.e. to reduce logic to grammar

- (A) Frege (1879, 1884): if we have logic, i.e. if we know which sentence entails which sentence, we have mathematics
- (B) Wittgenstein (1921): if we have grammar, i.e. if we know what a sentence is, we have logic (and solve all philosophical problems)

The Picture Theory of Language

- | | | |
|------|--|--|
| 4.01 | Der Satz ist ein Bild der Wirklichkeit [...] | A proposition is a picture of reality [...] |
| 3.21 | Der Konfiguration der einfachen Zeichen im Satzzeichen entspricht die Konfiguration der Gegenstände in der Sachlage. | To the configuration of the simple signs in the propositional sign corresponds the configuration of the objects in the state of affairs. |

Spatial objects

- | | | |
|--------|---|--|
| 4.014 | Die Grammophonplatte, der musikalische Gedanke, die Notenschrift, die Schallwellen, stehen alle in jener abbildenden internen Beziehung zu einander, die zwischen Sprache und Welt besteht. | The gramophone record, the musical thought, the score, the waves of sound, all stand to one another in that pictorial internal relation, which holds between language and the world. |
| 3.1431 | Sehr klar wird das Wesen des Satzeichens, wenn wir es uns, statt aus Schriftzeichen, aus räumlichen Gegenständen (etwa Tischen, Stühlen, Büchern) zusammengesetzt denken) | The essential nature of the propositional sign becomes very clear when we imagine it made up of spatial objects (such as tables, chairs, books) instead of written signs. |

Comparison

Let John be symbolized by a pebble ■, Mary by a marble ●, and smoking by a jar ○, where the fact that x smokes is expressed by placing the symbol for x inside the symbol for smoking.

Predicate Logic	PTL
$S(j)$	○■
$\neg S(m)$	○●
$S(j) \wedge \neg S(m)$	○■●

- (A) PTL has no logical constants
- (B) Consequently, inference is 'shown' and cannot be 'stated'

Logical constants

- 4.0312 [...] Mein Grundgedanke ist, dass die 'logischen Konstanten' nicht vertreten. Dass sich die Logik der Tatsachen nicht vertreten lässt.
- [...] My fundamental thought is that the 'logical constants' do not represent. That the logic of the facts cannot be represented.

Rules of inference

- | | | |
|-------|---|--|
| 5.13 | Dass die Wahrheit eines Satzes aus der Wahrheit anderer Sätze folgt, ersehen wir aus der Struktur der Sätze. | That the truth of one proposition follows from the truth of other propositions can be seen from the structure of the propositions. |
| 5.132 | ‘Schlussgesetze’, welche – wie bei Frege und Russell – die Schlüsse rechtfertigen sollen [...] wären überflüssig. | ‘Laws of inference’, which are supposed – e.g. by Frege and Russell – to justify inferences [...] would be superfluous. |
| 5.473 | Die Logik muss für sich selber sorgen [...] | Logic must take care of itself [...] |

Prediction

There is no way to translate $S(j) \wedge \neg S(j)$ into PTL: it is not possible to place to pebble inside and outside of the jar!

Let us conjecture that if there is no translation of ϕ into PTL, there is no translation of $\neg\phi$ into PTL either.

This means that trivialities are not sentences of PTL.

Trivialities

- | | | |
|-------|---|--|
| 4.462 | Tautologie und Kontradiktion sind nicht Bilder der Wirklichkeit [...] | Tautology and contradiction are not pictures of reality [...] |
| 4.466 | [...] Sätze, die für jede Sachlage wahr sind, können überhaupt keine Zeichenverbindungen sein [...] | [...] propositions that are true for every state of affairs cannot be combinations of signs at all [...] |

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Points made

- (A) Empirical and conceptual questions beset Abstraction (i.e. the thesis that well-formedness cannot be identified with meaningfulness)
- (B) Modern linguistic theory is consistent with Identity (i.e. the thesis that well-formedness is meaningfulness)
- (C) Identity is challenged by Logicality (i.e. the thesis that trivialities are non-sentences)
- (D) Wittgenstein's Picture Theory of Language provides a possible way to reconcile Logicality with Identity.

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- Abrusán, Martá. 2007. Contradiction and Grammar: The Case of Weak Islands. Doctoral Dissertation, MIT.
- Abrusán, Márta. 2019. Semantic anomaly, pragmatic infelicity, and ungrammaticality. Annual Review of Linguistics 5:329–351.
- Bar-Hillel, Yehoshua. 1957. Husserl's conception of a purely logical grammar. Philosophy and Phenomenological Research 17:362–369.
- Carnap, Rudolf. 1931. Überwindung der Metaphysik durch logische Analyse der Sprache. Erkenntnis 2:219–241.
- Carnie, Andrew. 2006. Syntax - A Generative Introduction. Blackwell Publishing, second edition.
- Chierchia, Gennaro. 2013. Logic in Grammar. Oxford: Oxford University Press.
- Chomsky, Noam. 1955. Logical syntax and semantics: Their linguistic relevance. Language 31:36–45.
- Chomsky, Noam. 1957. Syntactic Structures. The Hague: Mouton.
- Chomsky, Noam. 1981. Lectures on Government and Binding. Dordrecht: Foris.
- Chomsky, Noam. 1986. Barriers. Cambridge: MIT Press.
- Chomsky, Noam. 1995. The Minimalist Program. Cambridge: MIT Press.
- Chomsky, Noam. 2000. Minimalist inquiries: The framework. In Step by step: Essays on minimalist syntax in honor of Howard Lasnik, ed. Roger Martin, David Michaels, and Juan Uriagereka, 89–156. MIT Press.
- Chomsky, Noam. 2001. Derivation by phase. In Ken Hale: A Life in Linguistics, ed. Michael Kenstowicz, 1–52. Cambridge, Massachusetts: MIT Press.
- Del Pinal, Guillermo. 2019. The logicity of language: A new take on triviality, “ungrammaticality”, and logical form. Noûs 53:785–818.

- Del Pinal, Guillermo. 2022. Logicity of Language: Contextualism versus Semantic Minimalism. Mind 131:381–427.
- Dummett, Michael. 1973. Frege: Philosophy of Language. Cambridge: Harvard University Press.
- von Fintel, Kai. 1993. Exeptive constructions. Natural Language Semantics 1:123–148.
- Fox, Danny. 2024. Diagnosing Tree Tops. MIT class handout.
- Fox, Danny, Tue Trinh, and Itai Bassi. 2024. New perspectives on speech acts in grammar. Manuscript in progress.
- Frege, Gottlob. 1879. Begriffsschrift: Eine der arithmetischen nachgebildete Formelsprache des reinen Denkens. Halle: Neubert.
- Frege, Gottlob. 1884. Die Grundlagen der Arithmetik. Breslau: Verlage Wilhelm Koebner.
- Gajewski, Jon. 2002. L-analyticity and natural language. Manuscript, MIT.
- Gazdar, Gerald. 1979. Pragmatics: Implicature, Presupposition and Logical Form. New York: Academic Press.
- Haida, Andreas, and Tue Trinh. 2020. Zero and triviality. Glossa: A journal of general linguistics 5:1–14.
- Heim, Irene, and Angelika Kratzer. 1998. Semantics in Generative Grammar. Oxford: Blackwell.
- Husserl, Edmund. 1901. Logische Untersuchungen II. Halle: Max Niemeyer Verlag.
- Husserl, Edmund. 1907. Die Idee der Phänomenologie: Fünf Vorlesungen. Den Haag: Martinus Nijhoff.
- Krifka, Manfred. 2001. Quantifying into question acts. Natural Language Semantics 9:1–40.
- Krifka, Manfred. 2015. Bias in Commitment Space Semantics: Declarative questions, negated questions, and question tags. Proceedings of SALT 25:328–345.

- Krifka, Manfred. 2019. Commitments and beyond. Theoretical Linguistics 45:73–91.
- Krifka, Manfred. forthcoming. Zur Negierbarkeit von epistemischen Modalen. In Grammatik und Pragmatik der Negation im Deutschen, ed. Laura Neuhaus. Berlin: De Gruyter.
- Lakoff, George. 1970. Linguistics and natural logic. Synthese 22:151–271.
- Miyagawa, Shigeru. 2012. Agreement beyond Phi. Cambridge: MIT Press.
- Miyagawa, Shigeru. 2022. Syntax in the Treetops. Cambridge: MIT Press.
- Pistoia-Reda, Salvatore. 2024. Senso e controsenso: Uno studio filosofico su grammatica e logica delle lingue naturali. Roma: Carocci.
- Pistoia-Reda, Salvatore, and Luca San Mauro. 2021. On logicity and natural logic. Natural Language Semantics 29:501–506.
- Pistoia-Reda, Salvatore, and Uli Sauerland. 2021. Analyticity and modulation – broadening the rescale perspective on language logicity. International Review of Pragmatics 13:1–13.
- Radford, Andrew. 2004. Minimalist Syntax - Exploring the Structure of English. Cambridge: Cambridge University Press.
- Ross, John Robert. 1970. On declarative sentences. In Readings in English Transformational Grammar, ed. Roderick A. Jacobs and Peter S. Rosenbaum, 222–272. Waltham: Ginn and Company.
- Sauerland, Uli, and Kazuko Yatsushiro. 2017. Remind-me presuppositions and speech-act decomposition: Evidence from particles in questions. Linguistic Inquiry 48:651–677.
- Stenius, Erik. 1967. Mood and language games. Synthese 17:254–274.
- Trinh, Tue. 2022. Three ways of referring to discourse participants in Vietnamese. Journal of the Southeast Asian Linguistics Society 15:221–230.

- Trinh, Tue. 2024a. Forms of address, performative prefixes, and the syntax-pragmatics interface. Journal of Pragmatics 228:17–30.
- Trinh, Tue. 2024b. A note on speech act recursion. Proceedings of the Olomouc Linguistics Colloquium 2023:165–175.
- Trinh, Tue, and Itai Bassi. 2023. Excursive questions. Open Linguistics 9:20220232.
- Trinh, Tue, and Hubert Truckenbrodt. 2018. The Participant-Pronoun Restriction: English and Vietnamese. Proceedings of NAFOSTED 5:317–321.
- Wiltschko, Martina. 2021. The Grammar of Interactional Language. Cambridge: Cambridge University Press.
- Wittgenstein, Ludwig. 1921. Logisch-philosophische Abhandlung. Annalen der Naturphilosophie 14:185–262.
- Wittgenstein, Ludwig. 1953. Philosophische Untersuchungen. Frankfurt am Main: Suhrkamp.