Some observations about ‘zero’

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Abstract

In this paper, we present quantitative data regarding some (novel) observations about sentences containing the numeral **zero**. We propose a tentative account of these observations, and discuss the implications it has for existing theories of exhaustification and L-Analyticity.

1 A novel observation

The numeral **zero** cannot be modified by the adverb **at least**.

(1) a. there are at least 2 students in the classroom
b. *there are at least 0 students in the classroom

We conducted an experiment on Amazon MTurk. 32 English speakers rated the naturalness of 4 sentences comparable to (1a) and (1b) on a 4-point scale ($= 128$ scores for each type of sentence). Figure 1 shows sentences with **at least 2** received the highest score 4 (‘natural’) by $\geq 50\%$ of all subjects, while sentences with **at least 0** received the two lowest scores 2 and 1 (‘weird’) by $\geq 50\%$ of all subjects. The difference in the means of the scores (3.4 v 2.0), depicted in Figure 2, is highly significant ($p < 2.2^{-16}$).

Figure 1: Boxplot of **at least 2** and **at least 0**

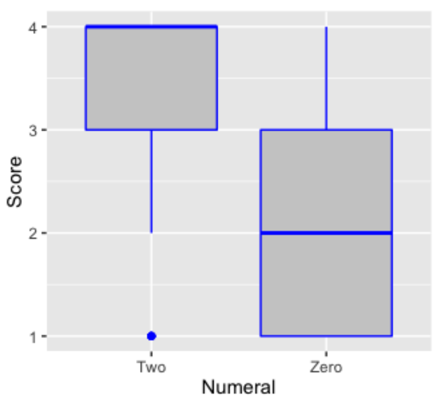
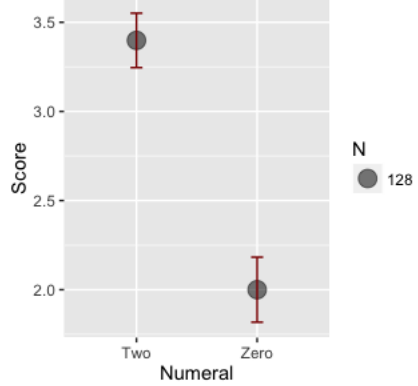


Figure 2: Means of **at least 2** and **at least 0**



2 Theoretical background

2.1 Exhaustification

Sentences are parsed with an exhaustifying operator **exh**$_C$ whose meaning is akin to that of the word **only**.

(2) $[\![\text{exh}_C\ \phi]\!] = 1$ iff $[\![\phi]\!] = 1 \wedge \forall\phi'\ [\phi' \in E^C_\phi \rightarrow [\![\phi']\!] = 0]$
i.e. $[\text{exh}_C\ \phi]$ is true iff ϕ is true and every excludable alternative of ϕ in C is false[1]

(3) $[_\psi\ \text{exh}_C\ [_\phi$ John talked to Mary or Sue$]]$

a. C = {$\underbrace{\text{John talked to Mary}}_{\text{non-excludable}}$, $\underbrace{\text{John talked to Sue}}_{\text{non-excludable}}$, $\underbrace{\text{John talked to Mary and Sue}}_{\text{excludable}}$}[2]

b. $[\![\psi]\!]$ = 1 iff John talked to Mary or Sue but not both

(4) $[_\psi\ \text{exh}_C\ [_\phi$ there are at least 2 students$]]$

a. C = {$\underbrace{\text{there are more than 2 students}}_{\text{non-excludable}}$, $\underbrace{\text{there are exactly 2 students}}_{\text{non-excludable}}$}[3]

b. $[\![\psi]\!] = [\![\phi]\!] = 1$ iff there are at least 2 students

2.2 Ignorance inferences

The sentences in (5) license "ignorance inferences" (Sauerland 2004, Geurts and Nouwen 2007, Fox 2007a, Buring 2008, Schwarz 2016).

(5) a. there are at least 2 students
$\rightsquigarrow \neg$K(exactly 2) $\wedge\ \neg$K($\neg$exactly 2)

b. John talked to Mary or Sue
$\rightsquigarrow \neg$K(mary) $\wedge\ \neg$K($\neg$mary)

2.2.1 Pragmatic derivation

The ignorance inference in question can be derived from the following generalization, itself a consequence of Grice's Maxim of Quantity and assumptions about relevance (Kroch 1972, Fox 2007a,b, Chierchia et al. 2012, Fox 2016, Buccola and Haida 2017).[4]

(6) Consequence of Quantity (CQ)
$[\text{exh}_C\ \phi]$ gives rise to the inference that the speaker does not know whether ϕ' is true, for every non-excludable ϕ' in C

(7) exh_C[there are at least 2 students]

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[1] Here is the formal definition: $E^A_\psi =_{\text{def}} \bigcap \{A' \mid A'$ is a maximal subset of A such that $\{\psi\} \cup \{\neg\psi' \mid \psi' \in A'\}$ is consistent$\}$. In this talk, we will not show how this definition derives the claims made in the text.

[2] For arguments that the alternatives of a disjunction are the individual disjuncts and the corresponding conjunction, see Sauerland (2004), Fox (2007a), Fox and Katzir (2011), Trinh and Haida (2015), Trinh (2018).

[3] For arguments that **at least n** alternates with **more than n** and **exactly n**, see Kennedy (2015), Buccola and Haida (2017).

[4] For lack of space, we will not present a derivation of CQ from the Maxim of Quantity and closure of relevance under negation.

a. C = $\{\underbrace{\text{more than 2}}_{\text{non-excludable}}, \underbrace{\text{exactly 2}}_{\text{non-excludable}}\}$

b. exh$_C$[... at least 2...] $\rightsquigarrow$ $\underbrace{\neg\text{K(more than 2)} \wedge \neg\text{K}(\neg\text{more than 2}) \wedge \neg\text{K(exactly 2)} \wedge \neg\text{K}(\neg\text{exactly}}_{\neg\text{K(exactly 2)} \wedge \neg\text{K}(\neg\text{exactly two})}$

2.2.2 Semantic derivation

It has, however, been claimed that ignorance inferences are to be derived semantically by way of the K operator (Meyer 2013, 2014, Buccola and Haida 2017). The logical form of (8) is assumed to be (9), in which exh$_C$ scopes over K, and the members of C contain K in their analysis.

(8) exh$_C$[K [there are at least 2 students]]

a. C = $\{\underbrace{\text{K [... more than 2 ...]}}_{\text{excludable}}, \underbrace{\text{K [... exactly 2...]}}_{\text{excludable}}\}$

b. $[\![$exh$_C$[K [... at least 2...]]$]\!] = 1$ iff $\underbrace{\text{K(at least 2)} \wedge \neg\text{K(exactly 2)} \wedge \neg\text{K(more than 2)}}_{\neg\text{K(exactly 2)} \wedge \neg\text{K}(\neg\text{exactly 2})}$

2.3 L-analyticity

Deviance may result from the sentence being "L-analytical," i.e. tautological or contradictory purely by virtue of the configuration of logical constants contained in it (Barwise and Cooper 1981, Fintel 1993, Gajewski 2003, Chierchia 2006, Abrusán 2007, Gajewski 2009, Abrusán 2011).

(9) a. there is a student $\Leftrightarrow \exists x[x \in S \wedge\ x \in E]$
b. *there is every student $\Leftrightarrow \forall x[x \in S \rightarrow x \in E] \Leftrightarrow_L \top$

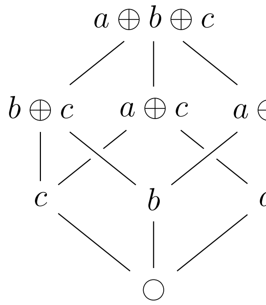
(10) a. everyone but Bill danced $\Leftrightarrow \forall x[x \notin \{b\} \rightarrow x \in D] \wedge \forall P[\forall x[x \notin P \rightarrow x \in D]$
b. *someone but Bill danced $\Leftrightarrow \exists x[x \notin \{b\} \wedge\ x \in D] \wedge \forall P[\exists x[x \notin P \wedge\ x \in D]$
$\rightarrow \{b\} \subseteq P]$
$\rightarrow \{b\} \subseteq P] \Leftrightarrow_L \bot$

Note that the sentences in (11) are not L-analytical, even though they are analytical. That is why they are not deviant.

(11) a. every bachelor is unmarried $\Leftrightarrow \top \not\Leftrightarrow_L \top$
b. there are students and there are no students $\Leftrightarrow \bot \not\Leftrightarrow_L \bot$

2.4 Semantics of 'zero'

Bylinina and Nouwen (2017) proposes that every plural noun has in its the denotation a special element, $\bigcirc$, whose cardinality is 0. Thus, suppose a, b and c are all and only the students in the world, then $[\![$students$]\!]$ would be the set containing all elements of the complete latice below.



(12) a. $[\![\text{n students}]\!] = [\lambda x\ [x \in [\![\text{students}]\!] \wedge \#x = n]]$
b. there are n students $\Leftrightarrow \exists x[x \in [\![\text{students}]\!] \wedge \#x = n]$

(13) a. $[\![\text{2 students}]\!] = [\lambda x\ [x \in [\![\text{students}]\!] \wedge \#x = 2]] = \{a \oplus b, b \oplus c, a \oplus c\}$
b. $[\![\text{0 students}]\!] = [\lambda x\ [x \in [\![\text{students}]\!] \wedge \#x = 0]] = \{\bigcirc\}$

(14) a. there are 2 students $\Leftrightarrow \exists x[x \in [\![\text{students}]\!] \wedge \#x = 2]$
b. there are 0 students $\Leftrightarrow \exists x[x \in [\![\text{students}]\!] \wedge \#x = 0] \Leftrightarrow_L \top$

Because <u>every</u> plural noun, by assumption, has $\bigcirc$ in its denotation, (14b) is L-analytical, although (14a) is not. However, L-analyticity can be circumvented by exhaustification: none of the sentences in (15) is L-analytical, assuming that every (unmodified) numeral alternates with every other numeral.

(15) a. $[_\psi$ exh$_C$ $[_\phi$ there are 2 students$]]$
$\Leftrightarrow \exists x[x \in [\![\text{students}]\!] \wedge \#x = 2] \wedge \neg\exists x[x \in [\![\text{students}]\!] \wedge \#x > 2]$
b. $[_\psi$ exh$_C$ $[_\phi$ there are 0 students$]]$
$\Leftrightarrow \exists x[x \in [\![\text{students}]\!] \wedge \#x = 0] \wedge \neg\exists x[x \in [\![\text{students}]\!] \wedge \#x > 0]$

Thus, sentences with **0** are always parsed with **exh**$_C$. This means that **0** must always mean ‘exactly 0’, while every other numeral **n** can mean either ‘at least n’ or ‘exactly n.’

(16) a. there are 2 students in the classroom, possibly more
b. #there are exactly 2 students in the classroom, possibly more

(17) a. #there are 0 students in the classroom, possibly more
b. #there are exactly 0 students in the classroom, possibly more

Zero doesn’t denote a negative generalized quantifier since it is neither downward entailing nor does it have the distribution of a generalized quantifier (Nouwen & Bylinina’s 2017):

(18) a. no/*zero students said anything
b. the number of students in the classroom is zero/*no
c. zero/*no students read the book, didn’t they

3 Deriving the observation

3.1 A simple account

Exhaustification is semantically inconsequential and therefore cannot obviate L-analyticity.

(19) #there are at least 0 students
a. $[_\psi$ exh$_C$ $[_\phi$ there are at least 0 students$]]$
b. $C = \{\underbrace{\text{there are more than 0 students}}_{\text{non-excludable}}, \underbrace{\text{there are exactly 0 students}}_{\text{non-excludable}}\}$
c. $\psi \Leftrightarrow \phi \Leftrightarrow \exists x[x \in [\![\text{students}]\!] \wedge \#x = 0] \Leftrightarrow_L \top$

3.2 First problem

Nouwen & Bylinina's (2017) theory assumes that exhaustification can rescue a sentence from L-analyticity. Abstracting from the problematic data regarding universal quantifiers, this assumption is crucial in explaining the contrast between (20a) and (20b).

(20) a. $[_{\psi \not\Leftrightarrow_L \top}$ exh$_C$ $[_{\phi \Leftrightarrow_L \top}$ there are 0 students$]]$
b. $[_{\psi \Leftrightarrow_L \top}$ exh$_C$ $[_{\phi \Leftrightarrow_L \top}$ there are at least 0 students$]]$

However, the deviance of (21) is evidence that the assumption is wrong.

(21) #there is every student

Assuming **every** alternates with **some**, the exhaustified meaning of (21) is 'there is no student,' which is not analytic.

(22) $[_\psi$ exh$_C$ $[_\phi$ there is every student$]]$
a. $\phi \Leftrightarrow \forall x[x \in S \rightarrow x \in E] \Leftrightarrow_L \top$
b. $\psi \Leftrightarrow \phi \wedge \neg \exists x[x \in S \wedge x \in E] \not\Leftrightarrow_L \top$

3.3 Second problem

Suppose the parse in (23a) is also available for (23).

(23) #there are at least 0 students
a. $[_\chi$ exh$_C$ $[_\psi$ K $[_\phi$ there are at least 0 students$]]]$
b. $C = \{\underbrace{[\text{K [there are more than 0 students]]}}_{\text{excludable}}, \underbrace{[\text{K [there are exactly 0 students]]}}_{\text{excludable}}\}$
c. $\chi \Leftrightarrow$ K(at least 0) $\wedge \neg$K(more than 0) $\wedge \neg$K(exactly 0) $\not\Leftrightarrow \top$

Thus, assuming K is represented in the syntax will lead to the wrong prediction, namely that there is a grammatical parse for (23), i.e. that (23) is not deviant.

It turns out that the problem is more general: the deviance persists with embedding of **at least** under any universal quantifier.

(24) a. #every human has at least 0 children
b. #you are required to read at least 0 books

3.4 A prediction

We expect a contrast between (25a) and (25b), since the latter can have a non-L-analytical parse.

(25) a. there are at least 0 students in the classroom
b. there are 0 or more students in the classroom

(26) there are 0 or more students
a. $[_\chi$ $[_\psi$ exh$_C$ $[_\phi$ there are 0 students$]]$ or $[_\omega$ there are more than 0 students$]]$
b. $\chi \Leftrightarrow \psi \vee \omega \Leftrightarrow \top \not\Leftrightarrow_L \top$

3.4.1 Empirical results for atomic sentences

In two experiments on Amazon MTurk, one a rating and one a forced choice experiment, we let participants adjudicate between seemingly synonymous sentences such as the pair in (27) with respect to how natural they sound.

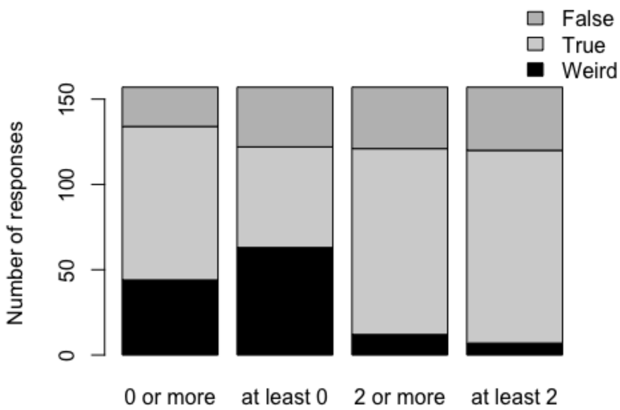
(27) a. *the company hired at least 0 employees
b. the company hired 0 or more employees

These two experiments failed to elicit responses that would substantiate our intuition, and the intuition of several colleagues we consulted with, that there is a contrast between the sentences in (27). However, a Google search of, e.g., the phrase **0 or more times** gives $170,000$ results, while **at least 0 times** only gives $2,780$ results.

3.4.2 Empirical results for universally quantified sentences

We conducted an experiment on Amazon MTurk (157 subjects, 1 judgment per subject and sentence) to substantiate the claim that **at least 0** has a different status from **0 or more**, **at least 2**, and **2 or more**. The proportion of 'weird' responses to **at least 0** is greater than that to its **0 or more** counterpart (40% and 28%, respectively, $p = 0.01605$). In contrast, the proportions of 'weird' responses to **at least 2** and **2 or more** are equal (7% and 12%, respectively, $p = 0.34$) (and smaller from **at least 0** and **0 or more**). See Figure 3 for illustration.

Figure 3: Number of True-False-Weird judgments



3.5 Are there better theories of zero?

We saw that the exhaustification account of the non-triviality of **there are zero students** is problematic. Therefore, let's assume that numerals have a two-sided meaning as a matter of semantic content (Breheny 2008, Geurts 2006, Kennedy 2015). We correctly derive that **there are zero students** is non-tautological:

(28) there are 0 students $\Leftrightarrow \max\{n \mid \exists x[x \in [\![\text{students}]\!] \wedge \#x = n]\} = 0$
$\Leftrightarrow \text{exh}_C$ [there are 0 students]
$\not\Leftrightarrow \top$

Moreover, we correctly derive that **there are at least zero students** is L-tautological:

(29) there are at least 0 students $\Leftrightarrow \max\{n \mid \exists x[x \in [\![\text{students}]\!] \wedge \#x = n]\} \geq 0$
$\Leftrightarrow \text{exh}_C$ [there are at least 0 students]
$\Leftrightarrow_L \top$

However, we still derive, incorrectly, that the deviance of **at least zero** is obviated under universal quantification:

(30) exh$_C$ [every human has at least 0 children]
$\Leftrightarrow \forall x[x \text{ is human} \rightarrow \max\{n \mid \exists y[x \text{ begot } y \wedge \#y = n]\} \geq 0]$
$\wedge\neg\forall x[x \text{ is human} \rightarrow \max\{n \mid \exists y[x \text{ begot } y \wedge \#y = n]\} = 0]$
$\wedge\neg\forall x[x \text{ is human} \rightarrow \max\{n \mid \exists y[x \text{ begot } y \wedge \#y = n]\} > 0]$
$\not\Leftrightarrow \top$

3.6 The logical status of scales

(31) a. #There are 0 students in the classroom
b. The temperature is at least 0 degrees Celsius
c. #The temperature is at least 0 degrees Kelvin

4 Conclusion

We conclude with a series of questions for future research.

- What is the semantics of **zero**?
- What is the semantics of **at least**?
- When can exhaustification obviate L-Analyticity (if ever)?

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